



BEAM OPTICS CALCULATIONS FOR A QUADRUPOLE CHANNEL
USING A PROGRAMMABLE DESK CALCULATOR

L. W. Oleksiuk

May 14, 1970

The purpose of this note is to inform interested parties of the existence of a desk calculator program developed by the author and useful for the design of quadrupole beam channels, with fast beam-design-computer interaction. The particular calculator used (Hewlett-Packard Model 9100A) includes an x-y plotter and printer peripherals all resident in the Beam Transfer Section. Programs to plot out relevant beam parameters are also available.

The Calculator:

The H-P calculator (Model 9100A) is a programmable desk calculator useful for simple engineering design problems. A read-write magnetic core memory provides a programming capacity of 196 machine instructions, coded as 6 bit words. Data storage is limited to five extra registers beyond the programming core. A program can be stored on a small magnetic card and re-entered into the memory at the user's convenience.

The Quadrupole Program →

The program calculates the usual beam parameters (β , α , γ) through an unlimited set of quadrupoles and drift spaces. Only one plane is calculated in a pass.

As each element in the channel is inputted, the beam parameters



are calculated, displayed and printed out.

Input:

The user enters the beam momentum and initial beam parameters, as the 1st and 2nd entries.

1st entry: = $\left[\begin{array}{l} \text{beam momentum in MoC units *31.30} \\ \text{magnetic rigidity (Kg-m)} \end{array} \right]$

2nd entry: = $\left[\begin{array}{l} \text{BETA} \quad \quad \quad (\text{metres}) \\ \text{ALPHA} \\ \text{initial beam parameters} \end{array} \right]$

3rd entry (and all succeeding entries) are the beam optical elements:

quadrupole length (m) drift length (m)
 or
 quadrupole gradient (Kg/metre) drift length (m)

To run the orthogonal plane calculations, simply input the appropriate beam beta and alpha functions, and reverse the signs of all the quadrupole gradients.

Extension to Bending Magnets:

Beam optic calculations involving standard gradient and wedge magnets can be performed using this program by utilizing the "effective gradient" parameters of these devices:

This parameter (in Kg/metre) and the magnet length essentially transform the device to an equivalent quadrupole of the same length.

Effective Gradient Parameters

Standard Gradient Magnet:

$$\left[\text{Gradient} \right] \text{ eff.} = \left[\frac{B_o^2}{(B\rho)} + G \right] \quad \text{bend plane}$$

$$\left[\text{Gradient} \right] \text{ eff.} = \left[-G \right] \quad \text{non-bend plane}$$

For a Wedge Magnet:

$$\text{Gradient eff.} = \left[\frac{B_o^2}{(B\rho)} \right] \quad \text{bend plane}$$

$$\text{Gradient eff.} = \left[0 \right] \quad \begin{array}{l} \text{non-bend plane} \\ \text{(input this element as} \\ \text{a drift space)} \end{array}$$

where $B_o \rightarrow$ Central field in Kg.

$(B\rho) \rightarrow$ beam magnetic rigidity in Kg-metres.

$G \rightarrow$ the quadrupole field component seen in the bend plane. (Kg/metre)

(G is not necessarily a positive number.)

Program: QUADRUPOLE CHANNEL

Operating Instructions:

A sample set of key operations for the Hewlett-Packard calculator is given below:

1. Set mode switch to RUN
2. Press [GO TO ()]
3. Press [0]
4. Press [0]
5. Set angle switch to RADIANS
6. Insert program card all the way into slot
7. Press [ENTER] button near slot

(The program is now in core.)

8. Press [CONTINUE]
9. Enter (beam momentum in MoC Units *31.30) into x register
10. Press [CONTINUE]
11. Enter initial [BETA function] in metres into x register
12. Press [↑]
13. Enter initial [ALPHA function] into x register
14. Press [CONTINUE]

(The printer will print out initial β α γ)

(Start optics loop here)

15. Enter [LENGTH] of element in metres
16. Press [↑]
17. Enter [GRADIENT] of element in Kg/metre

(for a drift space, skip Step 17)

18. Press [CONTINUE]

At this point, the printer will print out the sequence:

```
Inputted element  [LENGTH]  metres
Inputted element  [GRADIENT] Kg/metre (or repeat length
                                     f    drift space)
Calculated new [BETA function] metres
Calculated new [ALPHA function]
```

To enter further elements, loop back to Step 15.

Special Precaution:

If a quadrupole's gradient (Kg/metre) is numerically equal to it's length (metres), change one of the least significant digits to break this equality.

For example:

```
enter  1.000001 Kg/metres
        1.000000 metres
not    1.000000 Kg/metres
        1.000000 metres
```

The program uses this inequality to differentiate between quadrupole and drift space elements, an expediency necessitated by the small memory capacity.

Rotated Pole Subroutine:

Thin lens effects of rotated pole edges for a non-gradient magnet can be incorporated into an optical system using the equation

$$\alpha_{out} = \frac{(-\tan\theta)}{+|\rho|} \cdot \beta_{in} + \alpha_{in}$$

where the plus sign is used in the non-bend plane

The following subroutine must be executed via the Keyboard:

At the next element entry point →

enter Bo(Kg) (magnet field strength)

press [↑]

enter θ (radians) (pole edge rotation)

press [tanx]

press [*]

press [a]

press [÷]

press [e]

press [*]

press [f]

press [\pm] $\left(\begin{array}{l} \text{-in the bend plane} \\ \text{+in the non-bend plane} \end{array} \right)$

press [y→()]

press [f]

press [RCL]

press [GO TO]
() ()

press [0]

press [7]

press [continue]

now enter next element in the system.